

## Exercise 32

Find an equation of the tangent line to the given curve at the specified point.

$$y = \frac{1+x}{1+e^x}, \quad \left(0, \frac{1}{2}\right)$$

### Solution

Start by finding the slope of  $y$  at  $x = 0$ . Evaluate the derivative using the quotient rule.

$$\begin{aligned} y' &= \frac{d}{dx} \left( \frac{1+x}{1+e^x} \right) \\ &= \frac{\left[ \frac{d}{dx}(1+x) \right] (1+e^x) - \left[ \frac{d}{dx}(1+e^x) \right] (1+x)}{(1+e^x)^2} \\ &= \frac{(1)(1+e^x) - (e^x)(1+x)}{(1+e^x)^2} \\ &= \frac{1 - xe^x}{(1+e^x)^2} \end{aligned}$$

Evaluate it at  $x = 0$ .

$$y'(0) = \frac{1}{4}$$

Therefore, the equation of the line with slope  $1/4$  that goes through  $(0, \frac{1}{2})$  is

$$y - \frac{1}{2} = \frac{1}{4}(x - 0).$$

